SOFT DECISION DECODING OF BLOCK AND LATTICE CODES

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SUMMARY

Maximum likelihood soft decision decoding of block and lattice codes has recently gained renewed interest. This is partially due to the major role of block and lattice codes in block–coded modulation techniques for band limited channels [1],[2], and in part because of the feasibility of practical implementation of such sophisticated decoders due to the recent rapid progress in VLSI and DSP (digital signal processor) technologies. Most notably, the decoding algorithms derived by Conway & Sloane [3], Be'ery & Snyders [4], and Forney [5] are considerably more efficient than results published earlier [6]–[9]. The decoding procedures in the foregoing papers are presented with the aid of different tools, namely tables in [3], Hadamard transform in [4] and trellises in [5]. Nevertheless, they rely on an identical idea: decoding each coset of a subcode of certain types, in combination with the Wagner rule [6] or a generalized Wagner rule [11], when appropriate. Thus we may use the term '***Coset–Decoding***' to identify the common approach of these methods. These decoders are practically implementable for various codes, like Reed–Muller codes [4],[5] and related lattice codes [5], binary [3]–[5],[10] and ternary [11],[12] Golay codes, and the related Leech lattice code [3],[5],[14],[15], Hamming and BCH codes [4],[11],[13], the non–linear Nordstrom–Robinson code [5],[16],[17], and many other lattice codes [5],[12].

The Wagner decoding rule, which applies to single parity–check binary codes, is perhaps the oldest maximum likelihood soft decision decoding algorithm. It states that an entry–by–entry hard–detection of the received word is to be followed, unless the number of 1 bits is already even, by inversion of the least reliable bit. Several suboptimal algorithms contain, or generalize in a sense, the concept of the Wagner rule. These include the algorithms of Silverman & Balser [6], Forney [18], Chase [19], and Berlekamp [20] for soft decoding of binary linear block codes. Conway & Sloane [21] utilized the Wagner rule for decoding the

checkerboard lattices D_n (the lattices analogous of single parity–check codes), which they then applied [3] for maximum likelihood decoding of other lattices. Multiple–check generalization of the Wagner rule was presented by Snyders & Be'ery for binary block codes [11] and for the general q–ary case [22], and its application within the framework of Coset–Decoding was also examined.

The fast Hadamard transform (FHT) was first utilized by Green [7] (see also [23, p. 419]) for soft decision decoding of first–order Reed–Muller codes in earth–space communication. Application of FHT for decoding maximum–length shift–register codes (M–sequence) was presented in [24] and [25]. Utilization of the Green–Machine concept for decoding binary block and convolutional codes was described by Clark and Davis [26], but their procedure is efficient only for low–rate codes. A more efficient utilization, along with formal derivation, of FHT soft decoders is given in [4] and their application in the context of Coset–Decoding (but without using this term) is demonstrated. FHT decoders for high–rates, convolutional, product and concatenated codes are derived in [27].

Utilization of trellises for soft decoding of linear binary block codes, by employing the Viterbi algorithm, was first introduced by Massey [8] and Wolf [9]. The trellises of [8] are defined with the aid of the generator matrix of the code and the related decoders are efficient for low–rate codes, whereas those of [9] are based on the parity–check matrix of the code and thus the related decoders are efficient for high–rate codes. Forney [5, Appendix] define a *'trellis oriented generator matrix'* which is the basis for his improved decoding algorithms for block and lattice codes. Muder [28] investigates the properties of the trellis oriented generator matrices, and presents bounds on the number of states in the related trellises.

A general Coset–Decoding principle, for maximum likelihood decoding of both block and lattice codes, is described in [3]. It consists of the following steps: (a) find a subspace C_s of the given code, a basis for C_s and a basis for the coset representatives; (b) given the received word, obtain the most likely codeword in each coset, using a decoder for C_s; and (c) compare the codewords obtained and choose the most likely one as the decoding result. The key for achieving efficient decoders is a proper selection of a maximal subcode that is still easily decodable. Coset–Decoding was implemented [3]–[5] with respect to subcodes that are spanned by codewords which are either zero–concurring (i.e. the ones nowhere overlap), or one–concurring (i.e. the ones nowhere overlap except for several positions, where ones of all codewords overlap). These subspaces are, in a sense, geometrically similar [3] or contractible [11] to universe (i.e. zero parity–check) codes and single parity–check codes, respectively. The generalization of [11] utilizes subspaces that are contractible to codes with λ check bits. These subspaces are spanned by codewords that are, so called, λ–concurring. Constructive methods for finding maximal sets of λ–concurring codewords is still an open problem for many codes [13].

In the talk highlights of the above mentioned history of soft decision decoding of block and lattice codes will be reviewed with the aid of some examples. The recent results and the new terminology of Coset–Decoding and λ–concurring codewords will be explained. The relations, the common basis and the differences between the existing approaches (tables, trellises, FHT) will be discussed. The connection between the trellis oriented generator matrix and the λ–concurring codewords will be explored in the talk. New results on more efficient decoders for Reed–Muller, Nordstrom–Robinson and Kerdock codes will be demonstrated. The talk will be concluded with a few comments regarding the controversy about the preferability of convolutional and trellis codes over block and lattice codes.

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